Uncertainty Analysis

of the

Thunder Scientific
Model 2500
Two-Pressure Humidity Generator

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Relative Humidity Uncertainty Analysis

Of The

Thunder Scientific Model 2500 Two-Pressure Humidity Generator

Relative Humidity Uncertainty Analysis
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1 Introduction

Described here is the Relative Humidity Uncertainty Analysis, following NIST Guideline 1297[1], for a Model 2500 Humidity Generator that utilizes the NIST developed and proven two-pressure humidity generation principle.[2,3] Generation of humidity in a system of this type does not require direct measurements of the water vapor content of the gas. Rather, the generated humidity is derived from the measurements of saturation and chamber pressures, and saturation and chamber temperatures.

2 Defining Equation

Relative Humidity in a two-pressure humidity generator is determined from the measurements of temperature and pressure only and is expressed by the formula[3]

$$RH = P_c/P_s * E_s/E_c * F_s/F_c * 100 \quad [1]$$

where P_c = Chamber Pressure,
P_s = Saturation Pressure,
E_s = Saturation Vapor Pressure at Saturation Temperature,
E_c = Saturation Vapor Pressure at Chamber Temperature,
F_s = Enhancement Factor at Saturation Temperature and Pressure,
F_c = Enhancement Factor at Chamber Temperature and Pressure,
100 = nominal saturator efficiency.

3 Uncertainty Components

In the mathematical analysis of this formula, weíll analyze the uncertainties due to each of the above ratios separately, then combine the uncertainties to obtain the total expanded uncertainty. We are therefore concerned with four specific categories of uncertainty, each of which may have associated uncertainty components.

- uncertainty contribution from the pressure ratio term P_c/P_s, which includes
  - measurement uncertainty
  - measurement hysteresis
  - measurement resolution
- uncertainty contribution from the vapor pressure ratio term E_s/E_c, which includes
  - measurement uncertainty
  - saturation vs. chamber temperature intercomparison uncertainty
  - measurement resolution
  - chamber temperature self heating
- uncertainty contribution from the enhancement factor ratio Fs/Fc
- uncertainty contribution from saturator efficiency

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3.1 Uncertainty in the Pressure Ratio, Pc/Ps

The pressure ratio term, P_c/P_s, in a two-pressure humidity generator is the major %RH determining factor since both the E_s/E_c and F_s/F_c ratios are nearly equal to 1. Under those conditions, RH ≅ Pc/Ps * 100. To determine the affect a small change in pressure has on the computed RH, the difference can be taken between the RH computed with the pressure uncertainty included, and the RH computed without this uncertainty. This in effect is the partial numeric differential of RH with respect to pressure, computed at that pressure. The equation for this becomes

$$u(P) = \pm\{[(P_c \pm \partial P_c)/(P_s \pm \partial P_s) * F_s/F_c * E_s/E_c * 100] - RH\} \quad [2]$$

where u(P) is designated as uncertainty in RH due to pressure
∂P_c = uncertainty in chamber pressure measurement
∂P_s = uncertainty in saturation pressure measurement

This equation may be simplified by substituting $F_s/F_c = 1$ and $E_s/E_c = 1$, as those two ratios will be dealt with later. Also note, with these simplifying assumptions, that $RH = P_c/P_s * 100$. Since the focus here is on differences between the ideal and the uncertainty induced values, the simplifying assumption remains valid for the remainder of this uncertainty analysis. Now, applying each of these substitutions results in

$$u(P) = \pm\{(P_c \pm \partial P_c)/(P_s \pm \partial P_s) - (P_c/P_s)\} * 100 \quad [3]$$

In the use of this formula, it may be necessary to apply values to only ∂P_c or ∂P_s while maintaining all others constant. Or it may be necessary to apply both simultaneously with careful application of sign. The method chosen in each instance depends upon the component of uncertainty being evaluated and related factors.

In determining components of uncertainty, there are several things to consider, such as measurement uncertainty, measurement hysteresis, and measurement resolution.

3.1.1 Measurement Uncertainty Components of Pressure

Measurement uncertainty components of pressure were analyzed from *as found* data of 10 separate Model 2500 humidity generators during their annual recalibrations. Each system was tested at no fewer than 3 points over the range of the individual pressure transducers, resulting in no fewer than 30 measurement results from which to compute statistical standard deviations.

For saturation pressures above 50 psia, one transducer measures the chamber pressure and a separate transducer measures the saturation pressure. In this mode of operation, the standard deviation from the desired mean values are:

Std dev = 0.039 psia for P<50 psia
Std dev = 0.089 psia for P>50 psia

Therefore, the uncertainties in chamber and saturation pressure measurements, ∂P_c and ∂P_s, are

∂P_c = 0.039 psia

∂P_s = 0.039 psia for P<50
= 0.089 psia for P>50

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3.1.1.1 Measurement Uncertainty due to Pressure when Ps>50 psia

Sample calculations of the pressure uncertainty contributions would go as follows. First assume conditions where the ambient pressure is 14.7 psia. Since the chamber can operate only at ambient pressure, then the chamber pressure is P_c=14.7. The individual %RH uncertainty contributions due to the pressure ratio term under these conditions are then written and analyzed numerically for the high range saturation pressures as

$$\begin{aligned} u(P_c) &= \pm\{(P_c \pm \partial P_c)/(P_s) - (P_c/P_s)\} * 100 \\ &= \pm\{\pm\partial P_c/P_s\} * 100 \\ &= \pm\{ \pm 0.039/50\} * 100 \\ &= \pm 0.078 \text{ \%RH (at } P_s\text{=50, \%RH= Pc/Ps*100 = 29.40)} \end{aligned} \quad [4]$$

$$\begin{aligned} u(P_s) &= \pm\{(P_c)/(P_s \pm \partial P_s) - (P_c/P_s)\} * 100 \\ &= \pm\{(14.7)/(50 \pm 0.089) - (14.7/50)\} * 100 \\ &= \pm 0.052 \text{ \%RH (at Ps=50, \%RH =29.40)} \end{aligned} \quad [5]$$

where $u(P_c)$ is RH uncertainty due to uncertainty in chamber pressure P_c
$u(P_s)$ is RH uncertainty due to uncertainty in saturation pressure P_s

Note that in the above equations, only one value was varied at a time. Because of this, and also the fact that the values will be squared before further use, the sign of the result is of no concern. Now, performing the same calculations at a saturation pressure of 100 psia results in

$$\begin{aligned} u(P_c) &= \pm\{\pm 0.039/100\} * 100 \\ &= \pm 0.039 \text{ \%RH (at } P_s\text{=100, \%RH=14.70)} \end{aligned}$$

$$\begin{aligned} u(P_s) &= \pm\{(14.7)/(100 \pm 0.089) - (14.7/100)\} * 100 \\ &= \pm 0.013 \text{ \%RH (at } P_s\text{=100, \%RH=14.70)} \end{aligned}$$

Performing the same calculations at a saturation pressure of 150 psia results in

$$u(P_c) = \pm 0.026 \text{ \%RH (at } P_s\text{=150, \%RH=9.80)}$$

$$u(P_s) = \pm 0.006 \text{ \%RH (at } P_s\text{=150, \%RH=9.80)}$$

Notice that as saturation pressure increases, %RH uncertainty decreases as expected.

3.1.1.2 Measurement Uncertainty due to Chamber Pressure when P_s<50 psia

For saturation pressures below 50 psia, a different measurement scheme is employed. Rather than using two separate transducers for measuring chamber and saturation pressures, only one transducer is used and is time shared between the chamber and saturator. While this approach reduces RH uncertainty, it complicates the analysis somewhat. First, any measurement deviation in this single transducer will simultaneously affect both the chamber and saturation pressure readings. So when accounting for this uncertainty, it should be applied equally to both the chamber and saturation pressures simultaneously, and both instances of it must contain the same sign and magnitude. Computing uncertainty due to chamber pressure uncertainty then becomes

$$u(P_c) = \pm\{(P_c + \partial P_c)/(P_s + \partial P_c) - (P_c/P_s)\} * 100 \quad [6]$$

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Computing uncertainty due to chamber pressure measurement at various saturation pressures between P_s=15.5 and 50 psia results in

$$u(P_c) = \pm\{(14.7 + 0.039)/(15.5 + 0.039) - (14.7/15.5)\} * 100$$
$$= \pm 0.013 \text{ \%RH (at } P_s\text{=15.5, \%RH=94.84)}$$

$$u(P_c) = \pm\{(14.7 + 0.039)/(20 + 0.039) - (14.7/20)\} * 100$$
$$= \pm 0.052 \text{ \%RH (at } P_s\text{=20, \%RH=73.50)}$$

$$u(P_c) = \pm\{(14.7 + 0.039)/(30 + 0.039) - (14.7/30)\} * 100$$
$$= \pm 0.066 \text{ \%RH (at } P_s\text{=30, \%RH=49.00)}$$

$$u(P_c) = \pm\{(14.7 + 0.039)/(40 + 0.039) - (14.7/40)\} * 100$$
$$= \pm 0.061 \text{ \%RH (at } P_s\text{=40, \%RH=36.75)}$$

$$u(P_c) = \pm\{(14.7 + 0.039)/(50 + 0.039) - (14.7/50)\} * 100$$
$$= \pm 0.055 \text{ \%RH (at } P_s\text{=50, \%RH=29.4)}$$

3.1.1.3 Measurement Uncertainty due to Saturation Pressure with P_s<50

The final component of pressure measurement uncertainty to account for when dealing with saturation pressures below 50 psia is the uncertainty due to saturation pressure. The same transducer is used for both the saturation and chamber pressure measurements, and some uncertainty of this transducer has already been accounted for in the analysis due to chamber pressure measurement uncertainty. Double counting of the uncertainty component associated with the saturation pressure measurement can be avoided estimating its weighted value based on the value of the reading. For instance, at 50 psia the entire uncertainty of ±0.039 psia should apply, but at lower pressures, the uncertainty in measurement should drop proportionately. The uncertainty should therefore be accounted for as a function of reading rather than a straight sum. This scaled uncertainty in pressure measurement is then estimated as

$$\partial P_s = 0.039/50 * P_s$$
$$= 0.00078 * P_s \text{ [for } P_s\text{<50 psia]}$$

The associated uncertainty formula will then be

$$u(P_s) = \pm\{(P_c)/(P_s \pm \partial P_s) - (P_c/P_s)\} * 100$$
$$= \pm\{(P_c)/(P_s \pm (0.00078*P_s)) - (P_c/P_s)\} * 100$$
$$= \pm 0.078*P_c/P_s \text{ or } \pm 0.00078*RH \quad [7]$$

Computing uncertainty due to saturation pressure measurement at various saturation pressures between P_s=15.5 and 50 psia (with Pc=14.7) results in

$u(P_s) = \pm 0.074$ %RH (at Ps=15.5, %RH =94.84)
$u(P_s) = \pm 0.057$ %RH (at Ps=20, %RH =73.50)
$u(P_s) = \pm 0.038$ %RH (at Ps=30, %RH =49.00)
$u(P_s) = \pm 0.029$ %RH (at Ps=40, %RH =36.75)
$u(P_s) = \pm 0.023$ %RH (at Ps=50, %RH =29.40)

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3.1.2 Uncertainty due to Pressure Hysteresis

When the low range pressure transducer is time shared as it is for saturation pressures below 50 psia, the transducer is also subject to some measurement hysteresis. For more than 98% of the time, the transducer monitors the saturation pressure (approximately 5 minutes). For less than 2% of the time (once every 5 minutes for approximately 5 seconds), the transducer monitors the chamber pressure. By this criteria, it is only the chamber pressure which is affected by hysteresis. Again the sign of the deviation is important since hysteresis will always tend to increase the apparent measured value of the chamber pressure. The equation for uncertainty due to hysteresis, u(H), is

$$u(H) = \pm\{(P_c+\text{Hysteresis})/(P_s) - (P_c/P_s)\}*100$$

The maximum amount of hysteresis is estimated as +0.1% of the measured difference between the saturation and chamber pressures, with a rectangular distribution. The full interval is believed to ride on only one side of the true value, rather than centered about its mean. Therefore, the full interval, rather than half interval, is used in the following computations

$$\begin{aligned}\text{Hysteresis} &= \{0.1\% * (P_s-P_c)\}/\sqrt{3} \\ &= 0.00058 * (P_s-P_c)\end{aligned}$$

So the uncertainty component due to hysteresis is then computed as

$$\begin{aligned}u(H) &= \pm\{(P_c + 0.00058*(P_s-P_c))/(P_s) - (P_c/P_s)\} * 100 \\ &= \pm 0.058(1-P_c/P_s) \quad \text{or} \quad \pm 0.00058(100-RH)\end{aligned} \qquad [8]$$

Computing uncertainty due to hysteresis at various saturation pressures between P_s=15.5 and 50 psia (with P_c=14.7) results in

$u(H) = \pm 0.003$ %RH (at P_s=15.5, %RH=94.84)
$u(H) = \pm 0.015$ %RH (at P_s=20, %RH=73.50)
$u(H) = \pm 0.030$ %RH (at P_s=30, %RH=49.00)
$u(H) = \pm 0.034$ %RH (at P_s=40, %RH=36.75)
$u(H) = \pm 0.041$ %RH (at P_s=50, %RH=29.40)

3.1.3 Uncertainty in Pressure Measurement Resolution

The Analog to Digital conversion process resolves 1 part in 25000 over the range of each of the pressure transducers. Based on a rectangular distribution of the half-interval of resolution, the uncertainty component of pressure resolution is then

$$\text{resolution}_p = (\text{TransducerRange})/25000*0.5/\sqrt{3}$$

$$= 0.00058 \text{ psia [for } P<50]$$
$$= 0.00174 \text{ psia [for } P>50]$$

Since this uncertainty is specific to each and every individual measurement taken, it must be considered separately for both the chamber and saturation pressure measurements, regardless of which transducer is being utilized for the given operating conditions. The equations for uncertainty due to chamber pressure measurement resolution, $u(R_{Pc})$, and saturation pressure measurement resolution, u(RPs), are similar to equations 4 and 5 and are shown as

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$$u(R_{Pc}) = \pm\{resolution_p/P_s\} * 100 \quad [9]$$

$$u(R_{Ps}) = \pm\{(P_c)/(P_s \pm resolution_p) - (P_c/P_s)\} * 100 \quad [10]$$

Computing the uncertainties due to pressure measurement resolution at chamber pressure of 14.7 psia, and over the saturation pressure range of 15.5 to 150 psia results in

$u(R_{Pc}) = \pm 0.004$ %RH (at P_s=15.5, %RH=94.84)
$u(R_{Pc}) = \pm 0.003$ %RH (at P_s=20, %RH=73.50)
$u(R_{Pc}) = \pm 0.003$ %RH (at P_s=30, %RH=49.00)
$u(R_{Pc}) = \pm 0.001$ %RH (at P_s=40, %RH=36.75)
$u(R_{Pc}) = \pm 0.001$ %RH (at P_s=50, %RH=29.40)
$u(R_{Pc}) = \pm 0.001$ %RH (at P_s=100, %RH=14.70)
$u(R_{Pc}) = \pm 0.000$ %RH (at P_s=150, %RH=9.80)

$u(R_{Ps}) = \pm 0.004$ %RH (at P_s=15.5, %RH=94.84) [low range]
$u(R_{Ps}) = \pm 0.002$ %RH (at P_s=20, %RH=73.50) [low range]
$u(R_{Ps}) = \pm 0.001$ %RH (at P_s=30, %RH=49.00) [low range]
$u(R_{Ps}) = \pm 0.001$ %RH (at P_s=40, %RH=36.75) [low range]
$u(R_{Ps}) = \pm 0.000$ %RH (at P_s=50, %RH=29.40) [low range]
$u(R_{Ps}) = \pm 0.001$ %RH (at P_s=50, %RH=29.40) [high range]
$u(R_{Ps}) = \pm 0.000$ %RH (at P_s=100, %RH=14.70) [high range]
$u(R_{Ps}) = \pm 0.000$ %RH (at P_s=150, %RH=9.80) [high range]

3.1.4 Summary of Uncertainty in the Pressure Ratio P_c/P_s

The standard uncertainty, $u_c(P_c/P_s)$, in the pressure ratio P_c/P_s is determined from the associated individual components previously shown.

$$u_c^2(P_c/P_s) = u^2(P_c) + u^2(P_s) + u^2(Rp_c) + u^2(Rp_s) + u^2(H)$$

Standard Uncertainty Components of Rh due to pressure at Various Saturation Pressures

Source	**Type**	**Term**	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
			15.5	**20**	**30**	**40**	**50**	**50**	**100**	**150**
Measurement	A	$u(P_c)$	0.013	0.052	0.066	0.061	0.055	0.078	0.039	0.026
Measurement	A	$u(P_s)$	0.074	0.057	0.038	0.029	0.023	0.052	0.013	0.006
Resolution	B	$u(Rp_c)$	0.004	0.003	0.001	0.001	0.001	0.001	0.001	0.000
Resolution	B	$u(Rp_s)$	0.004	0.005	0.001	0.001	0.000	0.001	0.000	0.000
Hysteresis	B	u(H)	0.003	0.015	0.030	0.034	0.041			
combined		$u_c(P_c/P_s)$	0.075	0.079	0.082	0.076	0.072	0.094	0.041	0.027

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3.2 Uncertainty in the Vapor Pressure Ratio, E_s/E_c

E_s and E_c are Saturation Vapor Pressures computed at the saturation temperature and chamber temperature respectively, using the equation of Wexler[4]. In a perfectly ideal two-pressure humidity generator, the saturation temperature and chamber temperature would be exactly the same, resulting in an ideal E_s/E_c ratio of 1.00 exactly. A calculated E_s/E_c ratio of 1.0 contributes nothing to the calculation of %RH. However, in a real system, some slight differences do exist between the saturation and chamber temperatures, providing the need for measurement of these temperatures.

The uncertainty in RH due to temperature can be determined in a manner similar to that of equation 2, with the underlying assumptions that $F_s/F_c = 1$, and $P_c/P_s * 100 = RH$. The formula for computing the contribution due to temperature is

$$u(T) = \pm\{P_c/P_s * (E_{[Ts \pm \partial Ts]})/(E_{[Tc \pm \partial Tc]}) * F_s/F_c\} * 100 - RH \pm \{(E_{[Ts \pm \partial Ts]})/(E_{[Tc \pm \partial Tc]}) - 1\} * RH \quad [11]$$

where $u(T)$ is designated as uncertainty in RH due to temperature
∂T_c = uncertainty in chamber temperature measurement
∂T_s = uncertainty in saturation temperature measurement
$E_{[Ts - \partial Ts]}$ = Saturation Vapor Pressure computed at the Saturation Temperature, T_s, when perturbed by the possible temperature uncertainty, ∂T_s
$E_{[Tc + \partial Tc]}$ = Saturation Vapor Pressure computed at the Saturation Temperature, T_c, when perturbed by the possible temperature uncertainty, ∂T_c.

The individual uncertainty components which must be examined are measurement uncertainty, uncertainty of vapor pressure equations, saturation vs. chamber temperature intercomparison uncertainty, measurement resolution, and self heating.

3.2.1 Measurement Uncertainty Components of Temperature

Since the temperatures are always nearly equal, and are computed as a ratio of the corresponding saturation vapor pressures, it can be easily seen that if there is no mismatch between the chamber and saturation temperatures, then the ratio becomes 1.0 exactly and there is no uncertainty contribution due to temperature. This could also be true even if the temperature measurement of the two probes was actually incorrect or in error, provided the relative difference between them was zero. So if both were in error, but indicated the same numeric value at the same equal temperature, then again there would be no uncertainty contribution. Therefore, the contribution of uncertainty to RH due to temperature measurement accuracy is considered insignificant.

3.2.2 Uncertainty of Vapor Pressure Equations

The equations used for computation of vapor pressure are those of Wexler[4]. While there is uncertainty associated with the use of these equations, vapor pressures are always computed in ratio to one another with temperatures nearly equal to each another. Under these circumstances, the individual vapor pressure values, while they may be in error, cause no significant uncertainty when taken as a ratio. Therefore, the contribution due to uncertainty in the vapor pressure equations is considered insignificant.

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3.2.3 Saturation vs. Chamber Temperature Intercomparison Uncertainty

While the actual measurement accuracy of the two temperature probes is of little concern, the ability of the chamber and saturation temperature probes to indicate the same measured value at the same temperature is important, and is termed the intercomparison uncertainty. Intercomparison uncertainty was analyzed from *as found* data of 11 separate Model 2500 humidity generators during their annual recalibrations. Each system was tested at no fewer than 3 points over the range of 0 to 70ºC, resulting in 47 intercomparison results (difference between the indicated saturation temperature and indicated chamber temperature) from which to compute statistical standard deviation. The standard deviation of the difference between the saturation and chamber temperatures over the stated temperature range is

$$\text{Std dev} = 0.00985^{\circ}C$$

Since it is the difference between the temperature probes that is of concern, then the above number need only be applied to one of the temperatures while maintaining the other constant. In this case, the saturation temperature is chosen arbitrarily as the one to perturbate, while maintaining the chamber temperature constant at the ideal value. The perturbation amount which represents the intercomparison uncertainty is simply the standard deviation, and is therefore

$$\partial T_s = 0.00985^{\circ}C$$

The RH uncertainty due to temperature intercomparison, $u(T_i)$, is then written as

$$u(T_i) = \pm \{(E_{[Ts + 0.00985]})/(E_{[Tc]}) - 1\} * RH \qquad [12]$$

and may now be computed at several different saturation (or system) temperatures.

$$\begin{aligned} u(T_i) &= \pm \{(E_{0 + 0.00985})/(E_0) - 1\} * RH \\ &= \pm \{611.6495 / 611.1533 - 1\} * RH \\ &= \pm 0.00081 * RH \text{ (at } T_s=T_c=0^{\circ}C) \end{aligned}$$

$$\begin{aligned} &= \pm \{E_{35.00985}/E_{35} - 1\} * RH \\ &= \pm \{5629.514 / 5626.447 - 1\} * RH \\ &= \pm 0.00055 * RH \text{ (at } T_s=T_c=35^{\circ}C) \end{aligned}$$

$$\begin{aligned} &= \pm \{E_{70.00985}/E_{70} - 1\} * RH \\ &= \pm \{31190.6 / 31177.31 - 1\} * RH \\ &= \pm 0.00043 * RH \text{ (at } T_s=T_c=70^{\circ}C) \end{aligned}$$

3.2.4 Uncertainty in Temperature Measurement Resolution

The analog to digital conversion process, which transforms probe resistance into digital values resolves to 0.01ºC. Based on a rectangular distribution of the half-interval, the uncertainty component of temperature resolution is then

$$\begin{aligned} \text{resolution}_t &= 0.01 * 0.5/\sqrt{3} \\ &= 0.0029 \end{aligned}$$

Since this uncertainty is specific to each and every individual measurement taken, it must be considered separately for both the chamber and saturation temperature measurements. The

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equations for uncertainty due to chamber temperature resolution, $u(R_{Tc})$, and saturation temperature resolution, $u(R_{Ts})$, are given as

$$u(R_{Tc}) = \pm \{(E_{Ts}/E_{Tc+0.0029}) -1\} * RH \quad [13]$$

$$u(R_{Ts}) = \pm \{(E_{Ts+0.0029}/E_{Tc}) -1\} * RH \quad [14]$$

The uncertainty components due to temperature resolution can now be computed at several temperatures using the above equations.

$u(R_{Tc}) = \pm 0.00024 * RH$ (at $T_s=T_c=0ºC$)
$u(R_{Tc}) = \pm 0.00016 * RH$ (at $T_s=T_c=35ºC$)
$u(R_{Tc}) = \pm 0.00013 * RH$ (at $T_s=T_c=70ºC$)

$u(R_{Ts}) = \pm 0.00024 * RH$ (at $T_s=T_c=0ºC$)
$u(R_{Ts}) = \pm 0.00016 * RH$ (at $T_s=T_c=35ºC$)
$u(R_{Ts}) = \pm 0.00013 * RH$ (at $T_s=T_c=70ºC$)

3.2.5 Uncertainty due to Self Heating of Chamber Temperature Probe

The chamber temperature probe is generally calibrated and checked in a well-stirred fluid bath, but used in air. There is the possibility of some self-heating associated with this measurement then that must be considered. The self-heating, with temperature measurements in ºC, is estimated to be 0.05% of reading. The equation for the temperature uncertainty of self-heating is then

$$\begin{aligned} SelfHeating &= 0.05\% * T_c / \sqrt{3} \\ &= 0.00029 * T_c \end{aligned}$$

The equation for RH uncertainty due to self heating of the chamber temperature probe is then expressed as

$$u(SH) = \pm \{(E_{Ts}/E_{1.00029*Tc}) -1\} * RH \quad [15]$$

Again, computing this at several temperatures results in

$u(SH) = \pm 0$ (at $T_s=T_c=0ºC$)
$u(SH) = \pm 0.00055 * RH$ (at $T_s=T_c=35ºC$)
$u(SH) = \pm 0.00087 * RH$ (at $T_s=T_c=70ºC$)

3.2.6 Summary of Uncertainty in the Saturation Vapor Pressure Ratio E_s/E_c

The standard uncertainty of RH due to temperature, $u_c(E_s/E_c)$, in the saturation vapor pressure ratio E_s/E_c is determined form the individual components previously shown, and are combined using the equation

$$u_c^2(E_s/E_c) = u^2(T_i) + u^2(R_{Tc}) + u^2(R_{Ts}) + u^2(SH)$$

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Standard Uncertainty Components of RH due to Temperature at Various Temperatures					
Source	**Type**	**Term**	**Temperature**		
			0	**35**	**70**
Ts-Tc Intercomparison	A	u(Ti)/RH	0.00081	0.00055	0.00043
Ts Resolution	B	u(RTs)/RH	0.00024	0.00016	0.00013
Tc Resolution	B	u(RTc)/RH	0.00024	0.00016	0.00013
Self Heating	B	u(SH)/RH	0.00000	0.00055	0.00087
combined		u(Es/Ec)/RH	0.00088	0.00081	0.00099

3.3 Uncertainty in the Enhancement Factor Ratio F_s/F_c

Enhancement factors are slight correction factors used to account for the non-ideal behavior of water vapor when admixed with other gases. The enhancement factor is dependent on both temperature and pressure and is computed with the equation of Greenspan[5]. In determining the uncertainty due to the enhancement factor ratio, the individual uncertainty components that must be evaluated are measurement uncertainty due to temperature and pressure, and uncertainty of the enhancement factor equations.

3.3.1 Measurement Uncertainty due to Temperature and Pressure

The enhancement factor ratio, F_s/F_c, varies insignificantly with small perturbations in temperature and pressure. Uncertainties calculated from the ratio of F_s/F_c are at least an order of magnitude less than the uncertainties computed from the terms P_c/P_s and E_s/E_c. Therefore, measurement uncertainty due to temperature and pressure is considered negligible for this evaluation.

3.3.2 Uncertainty of the Enhancement Factor Equation

For ease of computation, the 2500 uses a simplified formula for enhancement factor ratio that is unique to this system, the results of which differ slightly from those of Greenspan[5]. The computational uncertainty of the enhancement factor ratio causes a corresponding uncertainty in computed RH of ±0.079% at 10%RH, reducing linearly toward an RH uncertainty of 0 at 100%. An equation to define this systematic uncertainty in RH due to the enhancement factor equation is written as

$$EqDiff = \pm[0.00088(100\text{-}RH)] \qquad [16]$$

Since this is determined to be a known, systematic, uncorrected error, the uncertainty in RH due to the enhancement factor equation, $u(F_{eq})$, is then

$$u(F_{eq}) = 0.00088(100\text{-}RH) \qquad [17]$$

3.3.3 Summary of Uncertainty in the Enhancement Factor Ratio

The standard uncertainty, $u_c(F_s/F_c)$, in the enhancement factor ratio is dominated by the uncertainty of the enhancement factor equation, and is therefore given as

$$u_c(F_s/F_c) = 0.00088(100\text{-}RH)$$

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Standard Uncertainty Components of RH due to Enhancement Factor										
Source	**Type**	**Term**	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
			15.5	**20**	**30**	**40**	**50**	**50**	**100**	**150**
			94.84 %RH	73.50 %RH	49.00 % RH	36.75 % RH	29.40 % RH	29.40 % RH	14.70 % RH	9.80% RH
Equation	B	$u(F_{eq})$	0.005	0.021	0.045	0.056	0.062	0.062	0.075	0.079
combined		$u_c(F_s/F_c)$	0.005	0.021	0.045	0.056	0.062	0.062	0.075	0.079

3.4 Uncertainty due to Saturator Efficiency

All two pressure humidity generators rely on the ability of the saturator to fully saturate the gas with water vapor as it passes from inlet to outlet. The efficiency of the saturation process is estimated as 100 ± 0.35% with triangular distribution about the mean value of 100. The standard uncertainty component due to saturator efficiency, when determined at the relative humidity being generated, is therefore computed as

$$u_c(SE) = 0.35/\sqrt{6} * RH = 0.143 * RH \quad [18]$$

4. Combined Standard Uncertainty

The combined standard uncertainty, $u_c(RH)$, is obtained by statistical combination of the standard uncertainty components of pressure ratio, vapor pressure ratio, enhancement factor ratio, and saturator efficiency. The combined uncertainty formula is then the sum of the variances

$$u_c^2(RH) = u_c^2(P_c/P_s) + u_c^2(E_s/E_c) + u_c^2(F_s/F_c) + u_c^2(SE) \quad [19]$$

The following tables reflect the standard uncertainty components and the combined standard uncertainty at various temperatures and pressures.

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Combined Standard Uncertainty of RH due to Standard Uncertainty Components at 0ºC										
Source	Type	Term	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
			15.5	20	30	40	50	50	100	150
			94.84% RH	73.50% RH	49.00% RH	36.75% RH	29.40% RH	29.40% RH	14.70% RH	9.80% RH
Pressure Ratio	A,B	$u_c(P_c/P_s)$	0.075	0.079	0.082	0.076	0.072	0.094	0.041	0.027
Vapor Pressure Ratio	A,B	$u_c(E_s/E_c)$	0.083	0.065	0.043	0.032	0.026	0.026	0.013	0.009
Enhancement Factor Ratio	B	$u_c(F_s/F_c)$	0.005	0.023	0.045	0.056	0.062	0.062	0.075	0.079
Saturator Efficiency	B	$u_c(SE)$	0.136	0.105	0.070	0.053	0.042	0.042	0.021	0.014
combined		$u_c(RH)$	0.176	0.148	0.124	0.113	0.107	0.123	0.089	0.085

Combined Standard Uncertainty of RH due to Standard Uncertainty Components at 35ºC										
Source	Type	Term	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
			15.5	20	30	40	50	50	100	150
			94.84% RH	73.50% RH	49.00% RH	36.75% RH	29.40% RH	29.40% RH	14.70% RH	9.80% RH
Pressure Ratio	A,B	$u_c(P_c/P_s)$	0.075	0.079	0.082	0.076	0.072	0.094	0.041	0.027
Vapor Pressure Ratio	A,B	$u_c(E_s/E_c)$	0.077	0.060	0.040	0.030	0.024	0.024	0.012	0.008
Enhancement Factor Ratio	B	$u_c(F_s/F_c)$	0.005	0.023	0.045	0.056	0.062	0.062	0.075	0.079
Saturator Efficiency	B	$u_c(SE)$	0.136	0.105	0.070	0.053	0.042	0.042	0.021	0.014
combined		$u_c(RH)$	0.173	0.146	0.123	0.112	0.110	0.123	0.089	0.085

Combined Standard Uncertainty of RH due to Standard Uncertainty Components at 70ºC										
Source	Type	Term	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
			15.5	20	30	40	50	50	100	150
			94.84% RH	73.50% RH	49.00% RH	36.75% RH	29.40% RH	29.40% RH	14.70% RH	9.80% RH
Pressure Ratio	A,B	$u_c(P_c/P_s)$	0.075	0.079	0.082	0.076	0.072	0.094	0.041	0.027
Vapor Pressure Ratio	A,B	$u_c(E_s/E_c)$	0.094	0.073	0.049	0.036	0.029	0.029	0.015	0.010
Enhancement Factor Ratio	B	$u_c(F_s/F_c)$	0.005	0.023	0.045	0.056	0.062	0.062	0.075	0.079
Saturator Efficiency	B	$u_c(SE)$	0.136	0.105	0.070	0.053	0.042	0.042	0.021	0.014
combined		$u_c(RH)$	0.182	0.152	0.127	0.114	0.108	0.124	0.089	0.085

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5 Expanded Uncertainty

Utilizing a coverage factor k=2, the expanded uncertainty, U, is expressed in the following table at various temperatures and humidities, using the formula

$$U = k * u_c(RH)$$

All values expressed for expanded uncertainty, U, are %Relative Humidity (%RH).

Expanded Uncertainty of RH with coverage factor k=2								
Saturation Temperature	Low Range Pressure, Ps<50					High Range Pressure, Ps>50		
	15.5	20	30	40	50	50	100	150
	94.84% RH	73.50% RH	49.00% RH	36.75% RH	29.40% RH	29.40% RH	14.70% RH	9.80% RH
0ºC	±0.35	±0.30	±0.25	±0.23	±0.21	±0.25	±0.18	±0.17
35ºC	±0.35	±0.29	±0.25	±0.22	±0.22	±0.25	±0.18	±0.17
70ºC	±0.36	±0.30	±0.25	±0.23	±0.22	±0.25	±0.18	±0.17

References:

1. Taylor, Barry N. and Kuyatt, Chris E.*, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994 Edition
2. Wexler, Arnold and Daniels, Raymond, *Pressure-Humidity Apparatus*, Journal of Research of the National Bureau of Standards, April 1952, Vol. 48, No. 4, 269-274.
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5. Greenspan, L., *Functional Equations for the Enhancement Factors for CO_2-Free Moist Air*, Journal of Research of the National Bureau of Standards - A. Physics and Chemistry, January-February 1976, Vol. 80A, No.1, 41-44
6. Kuyatt, Chris, et al., *Determining and Reporting Measurement Uncertainties*, Recommended Practice RP-12, National Conference of Standards Laboratories, April 1995

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Chamber Temperature Uncertainty Analysis

Of The

Thunder Scientific Model 2500 Two-Pressure Humidity Generator

Chamber Temperature Uncertainty Analysis
of the
Thunder Scientific Model 2500 Two-Pressure Humidity Generator

1 Introduction

Described here is the Chamber Temperature Uncertainty Analysis, following NIST Guideline 1297[1], for a Model 2500 Humidity Generator. The chamber temperature is measured with a 10k ohm thermistor, calibrated in-circuit against a reference thermometer in a well stirred fluid bath.

2 Defining Equation

The actual equation used to convert resistance of the thermistor to temperature is considered insignificant to this analysis since the thermistor is calibrated in the system, as a system to align the thermistorís indicated temperature readings with the reference thermometer. The exact equations and mathematics used to achieve this alignment are not considered in this analysis.

3 Uncertainty Components

In the mathematical analysis of chamber temperature, there are several factors to consider. Those factors include measurement uncertainty, measurement resolution, self heating, and uncertainty of the reference standard.

3.1 Measurement Uncertainty

For computation of chamber temperature uncertainty due to measurement uncertainty, analysis was performed on ***as found*** data of 10 separate Model 2500 humidity generators during their annual recalibrations. This data is from customer owned units, returned to Thunder Scientific for calibration, each with one year or more of service since the previous calibration***. This analysis typifies expected uncertainty after one year of in field use***.

Each chamber temperature was tested against a reference thermometer at 3 points over the range of 0 to 70ºC, resulting in 30 points from which to compute statistical standard deviation. The standard deviation of the difference between the reference standard and chamber temperatures over the stated temperature range is

$$\text{Std dev} = 0.018ºC$$

The uncertainty in chamber temperature due to measurement, u(M), is then the standard deviation of the repeated measurements just stated.

$$u(M) = 0.018ºC$$

3.2 Uncertainty in Temperature Measurement Resolution

The analog to digital conversion process which transforms probe resistance into digital values resolves to 0.01∞C. Based on a rectangular distribution of the half-interval, the uncertainty component of temperature resolution is then

$$u(R) = 0.01 * 0.5/\sqrt{3}$$
$$= 0.0029$$

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3.3 Uncertainty due to Self Heating of Chamber Temperature Probe

The chamber temperature probe is generally calibrated and checked in a well stirred fluid bath, but used in air. There is the possibility of some self heating associated with this measurement then that must be considered. The self heating, with temperature measurements in ºC, is estimated to be +0.05% of reading. Based on rectangular distribution of the interval, the equation for the temperature uncertainty of self heating, u(SH), is then

$$u(SH) = 0.05\% * T_c / \sqrt{3}$$
$$= 0.00029 * T_c$$

3.4 Uncertainty of the Temperature Reference Standard

The reference thermometer has a manufacturer stated accuracy of ±0.01ºC. Assuming rectangular distribution of the half interval, the uncertainty of the temperature reference standard, $u(T_{ref})$, is then

$$u(T_{ref}) = 0.01/\sqrt{3}$$
$$= 0.006ºC$$

4 Combined Standard Uncertainty of Chamber Temperature

The standard uncertainty components and the resulting combined standard uncertainty of chamber temperature, $u_c(T_c)$, are listed in the following table. The combined uncertainty was computed as the square root of the sum of the variances with the equation

$$u_c^2(T_c) = u^2(M) + u^2(R) + u^2(SH) + u^2(T_{ref})$$

Standard Uncertainty Components of Chamber Temperature					
Source	**Type**	**Term**	**Temperature**		
			0	**35**	**70**
Measurement	A	u(M)	0.018		
Resolution	B	u(R)	0.003		
Self Heating	B	u(SH)	0.000	0.010	0.020
Reference	B	$u(T_{ref})$	0.006		
combined		$u_c(T_c)$	.019	.022	.028

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5. Expanded Uncertainty

Utilizing a coverage factor k=2, the expanded uncertainty, U, is listed in the following table at various temperatures using the following formula.

$$U = k * u_c(T_c)$$

Expanded Uncertainty of Chamber Temperature with Coverage Factor k=2					
Source	**Term**	**Time Interval**	**Temperature**		
			0	**35**	**70**
Chamber Temperature	U	One Year	±0.038ºC	±0.044ºC	±0.056ºC

Note that the expanded uncertainties shown represent expected uncertainties after one year of use.

References:

1. Taylor, Barry N. and Kuyatt, Chris E.*, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994 Edition
2. Kuyatt, Chris, et al., *Determining and Reporting Measurement Uncertainties*, Recommended Practice RP-12, National Conference of Standards Laboratories, April 1995

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Chamber Temperature Uniformity Analysis

Of The

Thunder Scientific Model 2500 Two-Pressure Humidity Generator

Chamber Temperature Uniformity Analysis
of the
Thunder Scientific Model 2500 Two-Pressure Humidity Generator

1 Introduction

Described here is the Chamber Temperature Uniformity for a Model 2500 Humidity Generator. Chamber temperature uniformity has a direct influence on relative humidity gradients within the test chamber. In order to determine the chamber temperature uniformity, 10 thermometers of equivalent type and nominal resistance where calibrated together over the temperature range 15 to 35ºC. The thermometers were then strategically placed at various locations within the test chamber, approximately 1 to 2 inches from each corner (8 probes total), and 2 inches left and right of center (2 probes total).

2 Defining Equations

The maximum measurement deviation from the mean will be determined by noting the maximum and minimum readings from the set of probes at the same point in time, then taking half the difference of these values.

$$\text{MaxDev} = \pm 0.5(\text{MaxReading}-\text{MinReading}) \quad [1]$$

The uniformity will then be computed by RSS combination (root of the sum of the squares) of the maximum deviation, MaxDev, and the estimated thermometer uncertainty, u(T).

$$\text{uniformity}^2 = \text{MaxDev}^2 + u^2(T) \quad [2]$$

3 Calibration of Thermometers

The 10 thermometers were calibrated at the same time, in the same bath, against the same reference thermometer. Although they were calibrated in a well stirred fluid bath, yet used in air, self heating is not considered a significant contributor since all probes are used in the same type of environment. All should be subjected to similar self heating effects which tend to cancel one another when viewing differences between probes. The accuracy of the reference standard is also considered insignificant, since the desired value here is relative probe difference, not individual probe accuracy. The only concern in calibration of the thermometers is the relative accuracy of each with respect to the group. With this in mind, the uncertainty of the probes, u(T), with respect to each other after calibration is estimated to be

$$u(T) = \pm 0.025ºC$$

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3.1 Measurement of Chamber Temperatures

The following data was gathered during the uniformity analysis conducted on 4 Dec 1997, using a Model 2500, serial number 9711116. The generator was run at a fixed humidity of 50% RH, and was allowed to stabilize for a minimum of one hour at each temperature listed. Note that the maximum and minimum readings are indicated in bold type.

Probe	Location	15ºC nominal	25ºC ambient	35ºC nominal
1	Lower Left Front	15.169	24.937	34.856
2	Lower Right Front	15.215	24.906	34.812
3	Lower Left Rear	**15.333**	**24.938**	**34.710**
4	Lower Right Rear	15.225	24.916	34.787
5	Upper Left Front	**15.150**	24.935	**34.868**
6	Upper Right Front	15.164	24.897	34.853
7	Upper Left Rear	15.163	24.935	34.850
8	Upper Right Rear	15.156	24.914	34.852
9	Left Center	15.171	24.917	34.847
10	Right Center	15.169	**24.894**	34.829
Maximum Deviation (MaxDev)		±0.0915	±0.022	±0.079

4. Chamber Temperature Uniformity

As per equation 2, the uniformity at each of the 3 temperatures is computed as

$$\text{uniformity} = \text{sqr}(0.0915^2 + 0.025^2)$$

$$= \pm 0.095ºC \text{ (at } 15ºC)$$
$$= \pm 0.033ºC \text{ (at } 25ºC)$$
$$= \pm 0.083ºC \text{ (at } 35ºC)$$

This is within the stated accuracy specification of ±0.10ºC when the chamber is operated within ±10ºC of ambient temperature.

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